Quadratic equations & inequations

[Class assignment, advance level]

Solution of quadratic equations

Advance Level

1. If $-1 \le x < 0$, then solution of the equation $|x+1|-|x|+3|x-1||x-2|=x+2$ is

(a) 1, 5/3 (b) 5/3 (c) 1/3 (d) None of these

2. The real roots of $|x|^3 - 3x^2 + 3|x| - 2 = 0$ are

(a) 0, 2 (b) ± 1 (c) ± 2 (d) 1, 2

3. The number of real solutions of the equation $2^{x/2} + (\sqrt{2}+1)^x = (5+2\sqrt{2})^{x/2}$ is

(a) One (b) Two (c) Four (d) Infinite

4. The number of negative integral solutions of $x^2.2^{x+1} + 2^{|x-3|+2} = x^2.2^{|x-3|+4} + 2^{x-1}$ is

(a) 0 (b) 1 (c) 2 (d) 4

5. The equation $e^x - x - 1 = 0$ has

(a) Only one real root $x = 0$
(b) At least two real roots
(c) Exactly two real roots
(d) Infinitely many real roots

6. The number of real roots of the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ are

(a) 1 (b) 2 (c) Infinite (d) None of these

7. If a, b, c are positive real numbers, then the number of real roots of the equation $ax^2 + b|x| + c = 0$ is

(a) 2 (b) 4 (c) 0 (d) None of these

8. The number of real solutions of equation $\log_{10}[98 + \sqrt{[x^3 - x^2 - 12x + 36]}] = 2$ are

(a) 4 (b) 1 (c) 2 (d) 3

9. The equation $x^{(3/4)(\log_2 x)^2 + (\log_2 x) - 5/4} = \sqrt{2}$ has

(a) At least one real solution
(b) Exactly three real solutions
(c) Exactly one irrational solution
(d) All the above

10. The number of solutions of $|[x] - 2x| = 4$, where $[x]$ is the greatest integer $\le x$, is

(a) 2 (b) 4 (c) 1 (d) Infinite

Nature of roots

Advance Level

11. Equation $\frac{a^2}{x-\alpha} + \frac{b^2}{x-\beta} + \frac{c^2}{x-\gamma} = m - n^2 x$ ($a, b, c, m, n \in R$) has necessarily

(a) All the roots real
(b) All the roots imaginary
(c) Two real and two imaginary roots
(d) Two rational and two irrational roots

12. If $\cos\theta$, $\sin\phi$, $\sin\theta$ are in G.P. then roots of $x^2 + 2\cot\phi\, x + 1 = 0$ are always

(a) Equal (b) Real
(c) Imaginary (d) Greater than 1

13. If $f(x)$ is a continuous function and attains only rational values and $f(0) = 3$, then roots of equation $f(1)x^2 + f(3)x + f(5) = 0$ are

(a) Imaginary (b) Rational
(c) Irrational (d) Real and equal

14. The roots of $ax^2 + bx + c = 0$, where $a \ne 0$ and coefficients are real, are non-real complex and $a + c < b$. Then

(a) $4a + c > 2b$ (b) $4a + c < 2b$
(c) $4a + c = 2b$ (d) None of these

15. The equation $(a+2)x^2 + (a-3)x = 2a - 1, a \ne -2$ has roots rational for

(a) All rational values of a except $a = -2$
(b) All real values of a except $a = -2$
(c) Rational values of $a > \frac{1}{2}$
(d) None of these

16. The quadratic equation $x^2 - 2x - \lambda = 0, \lambda \ne 0$

(a) Cannot have a real root if $\lambda < 1$

(b) Can have a rational root if λ is a perfect square

(c) Cannot have an integral root if $n^2 - 1 < \lambda < n^2 + 2n$ where $n = 0, 1, 2, 3, ...$

(d) None of these

17. If the roots of the equation $x^2 + px + q = 0$ are α and β and roots of the equation $x^2 - xr + s = 0$ are α^4, β^4, then the roots of the equation $x^2 - 4qx + 2q^2 - r = 0$ will be **[IIT 1989]**

(a) Both negative (b) Both positive

(c) Both real (d) One -ve & one +ve

18. If equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, $a, b, c > 0$, $n \in N$, then

(a) $a^n + c^n \geq 2b^n$ (b) $a^n + c^n > 2b^n$

(c) $a^n + c^n \leq 2b^n$ (d) $a^n + c^n < 2b^n$

19. If $\dfrac{\sum_{r=0}^{k-1} x^{2r}}{\sum_{r=0}^{k-1} x^r}$ is a polynomial in x for two values of p and q of k, then roots of equation $x^2 + px + q = 0$ cannot be

(a)Real (b) Imaginary (c) Rational (d) Irrational

20. If $x > 0$, $f(x) = (a - x^n)^{1/n}$, $g(x) = x^2 + px + q, p, q \in R$ and equation $g(x) - x = 0$ has imaginary roots, then number of real roots of equation $g(g(x)) - f(f(x)) = 0$ is

(a) 0 (b) 2 (c) 4 (d) None of these

Advance-level answer sheet:

1	2	3	4	5	6	7	8	9	10
d	c	a	a	a	d	c	b	d	b
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
d	a	a	b	a	b	a	a,c	c	b