Alternating Current: Answer

ANSWER KEYS

SINGLE CORRECT ANSWER TYPE

LEVEL - I

1.	(c)	2.	(b)	3.	(c)	4.	(a)
5.	(a)	6.	(d)	7.	(d)	8.	(c)
9.	(c)	10.	(a)	11.	(a)	12.	(d)
13.	(b)	14.	(a)	15.	(c)	16.	(b)
17.	(a)	18.	(c)	19.	(d)	20.	(b)
21.	(a)	22.	(b)	23.	(b)	24.	(b)
25.	(d)	26.	(b)	27.	(d)	28.	(b)
29.	(b)	30.	(b)	31.	(d)	32.	(c)
33.	(c)	34.	(b)	35.	(c)	36.	(b)
37.	(c)	38.	(a)	49.	(d)	40.	(c)
41.	(a)						

LEVEL - II

42.	(b)	43.	(d)	44.	(d)	45.	(d)
46.	(a)	47.	(c)	48.	(d)	49.	(a)
50.	(d)	51.	(d)	52.	(a)	53.	(d)
54.	(c)	55.	(b)	56.	(d)	57.	(c)
58.	(d)	59.	(c)	60.	(c)	61.	(a)
62.	(c)						

LEVEL - III

63.	(c)	64.	(d)	65.	(a)	66.	(a)
67.	(b)	68.	(a)	69.	(b)	70.	(d)

MULTIPLE CORRECT ANSWERS TYPE

LEVEL - I

1.	(a), (b), (d)	2.	(a),(c),(d)	3.	(a) (b)	4.	(c) (d)
5.	(a), (b)	6.	(b) (d)	7.	(a)	8.	(a), (d)
9.	(a), (d)						

LEVEL - II

10.	(a), (d)	11.	(b), (c), (d)	12.	(a), (b), (d)
13.	(a), (b), (c)	14.	(a), (c), (d)	15.	(d)
16.	(a)	17.	(d)		

LEVEL - III

18.	(a), (b), (c)	19.	(a),(c)	20.	(a),(b)	21.	(a), (b), (c), (d)
22.	(a), (b), (c)	23.	(a), (b)				

COMPHREHENSIVE TYPE QUESTIONS

1.	(a)	2.	(c)	3.	(c)	4.	(a)	5.	(c)
6.	(a)	7.	(b)	8.	(a)	9.	(a)	10.	(a)
11.	(a)	12.	(c)						

MATRIX MATCHING TYPE QUESTIONS

1. $A \rightarrow (P,S,T)$, $B \rightarrow (P,S,T)$, $C \rightarrow (P,R)$, $D \rightarrow (P,Q)$
2. $A \rightarrow (R)$, $B \rightarrow (P)$, $C \rightarrow (S)$, $D \rightarrow (Q, S)$
3. $A \rightarrow (P,S)$, $B \rightarrow (Q)$, $C \rightarrow (P,R)$, $D \rightarrow (P,S)$
4. $A \rightarrow (R)$, $B \rightarrow (S)$, $C \rightarrow (P)$, $D \rightarrow (Q)$
5. $A \rightarrow (Q)$, $B \rightarrow (S)$, $C \rightarrow (Q, R)$, $D \rightarrow P$
6. $A \rightarrow (Q,R)$, $B \rightarrow (P,S)$, $C \rightarrow (P,R)$, $D \rightarrow (Q,S)$
7. $A \rightarrow (S)$, $B \rightarrow (P,R,S)$, $C \rightarrow (Q,S,T)$, $D \rightarrow (Q)$

ASSERTION REASONING TYPE QUESTIONS

1. (B) 2. (C) 3. (A) 4. (A)
5. (A) 6. (A)

INTEGER TYPE QUESTIONS

1	2	3	4	5	6	7	8
1	**1**	1	1	1	1	1	1
2	2	**2**	2	2	**2**	**2**	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	**5**	5	5	**5**
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	**8**	8	8	8	8
9	9	9	9	9	9	9	9

SUBJECTIVE QUESTIONS

1. (a) 8.24 A (b) V_L=207V, V_C=437 (c) zero (d) zero (e) zero
2. (a) 663 Hz, I_{Max}=14.1A (b) 663 Hz, P_{max}=2300W
   (c) 648 Hz and 678 Hz, 10A (d) Q=21.7
3. $\sqrt{\frac{3}{5}}$
4. 75° (leading)
5. 0.386 amp
6. 0.29 amp, $\cos\theta = 0.8$
7. $15^o 5', 710.4\, watt$
8. (a) 1.0 J, yes, (b) w = 10^3 rad/sec
   (c) (i) $t = 0, \frac{T}{2}, T, \frac{3T}{2}, ...$ (ii) at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, ...$
   (d) At $T = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, ...$ (e) 1.0 Joules

9. (i) 4mf (ii) 2.53 H (iii) 720 Ω

10. $L = 0.08H$, $P_{av} = 27W$, $f_0 = 11.25Hz$

11. P.F = 0.8, P_{av} = 74.88

12. $R = 9.8\sqrt{2}\Omega$ $I_0 = 7.22A$

13. $\frac{\pi}{4}$

14. 2.5 to 250 pico farad

15. 1.2H

16. $75\mu F$

17. $Irms = \frac{8I_0^{\ 2}}{T^3}$

PREVIOUS IIT JEE QUESTIONS

1. (a) 10^4 A/s (b) zero (c) 2.0 A (d) 1.732×10^{-4} *C*

2. (a)

3. 20 A, $\frac{\pi}{4}$

HINTS AND SOLUTIONS

SINGLE CORRECT ANSWER TYPE
LEVEL - I

1. I = $4\sin\left(100\pi t + \frac{\pi}{6}\right)$Amp

Initial value of current $= I_{t=0} = 4\sin\left(100\pi \times 0 + \frac{\pi}{6}\right)$

$= 4\sin\frac{\pi}{6} = 4 \times \frac{1}{2} = 2$ Amp

2. Power loss = 20 watt

Output power = VI = $220 \times 4 = 880$ watt

Power generated by dynamo = 900 Watt

$\therefore$ EI = 900 $\Rightarrow E = \frac{900}{I} = \frac{900}{4} = 225$ volt

3. $R = \frac{E - V}{I} = \frac{120 - 115}{25} = 0.2\Omega$

4. Compare with

$i = i_p \sin(\omega t + \phi) = i_p \sin\omega t \cos\phi + i_p \cos\omega t \sin\phi$

Thus $i_p \cos\phi = 10, i_p \sin\phi = 8$. Hence $\tan\phi = \frac{4}{5}$

5. $I = \frac{1}{\sqrt{2}} \sin 314t$

$E = \sqrt{2} \sin\left(314t - \frac{\pi}{6}\right)$

with respect to current, the emf is lagging.

So phase difference is $\phi = -\frac{\pi}{6}$

6. $I_{rms} = \frac{I_o}{\sqrt{2}} = \frac{14}{\sqrt{2}} = 10\,A$

To get same heating effect the constant current should be 10 A.

7. $I_{rms} = 10A \Rightarrow I_o = 10\sqrt{2}\,A = 14.14\,Amp$

Let I = 0 when t = 0.

$\therefore \; I = I_o \sin\omega t$

$I_o = I_o \sin\omega t \quad \Rightarrow \omega t = \frac{\pi}{2}$

$\Rightarrow t = \frac{\pi}{2\omega} = \frac{\pi}{2 \times 2\pi \times f} = \frac{1}{200} = 5 \times 10^{-3}\ \text{sec}$

8. $V_{rms} = 12\,V \quad \Rightarrow V_o = 12\sqrt{2}$

RMS value of AC voltage is equivalent to DC voltage.

9. $i^2 = 9t^2$

$$< i^2 >_{0-1} = \frac{\int_0^1 9t^2 dt}{\int_0^1 dt} = 3$$

$\therefore \quad i_{rms} = \sqrt{3}\ A$

10. In case of a coil, i.e., L-R circuit,

$i = \frac{V}{Z}$ with $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$

So when dc is applied, $\omega = 0$, so Z = R.

and hence $i = \frac{V}{R}$ i.e. $R = \frac{V}{I} = \frac{100}{0.5} = 200\,\Omega$

But $\quad Z = \sqrt{R^2 + \omega^2 L^2}$ i.e. $\omega^2 L^2 = Z^2 - R^2$

i.e. $\quad (2\pi f L)^2 = 200^2 - 100^2 = 3 \times 10^4 \quad (\text{as } \omega = 2\pi f)$

So, $\quad L = \frac{\sqrt{3} \times 10^2}{2\pi \times 50} = \frac{\sqrt{3}}{\pi} H = 0.55\,H$

11. A solenoid consists of inductance and resistance.

When 100 V dc is applied, $\omega = 0 \Rightarrow Z = R$

$$Z = \frac{V_{rms}}{I_{rms}} \Rightarrow R = \frac{100}{1} = 100\,\Omega$$

When 100 V, 50 Hz ac is applied,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{100}{0.5} = 200\,\Omega$$

$$Z^2 = R^2 + X_L^2 \Rightarrow 200^2 = (100)^2 + X_C^2$$

$$\Rightarrow X_L = -100\sqrt{3} \Rightarrow 2\pi fL = 100\sqrt{3} \Rightarrow L = \frac{100\sqrt{3}}{2\times\pi\times 50} = 0.55\,H$$

12. $X_L = 2\pi fL = 2\pi \times 200 \times \frac{1}{\pi} = 400\,\Omega$

$R = 300\Omega$

$$\tan\phi = \frac{X_L}{R} = \frac{400}{300} = \frac{4}{3}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{4}{3}\right)$$

13. $X_C = \frac{1}{\omega C} = \frac{1}{100\times 10^{-6}} = 10^4\,\Omega$

AC ammeter gives rms value of current.

$$I_{rms} = \frac{E_{rms}}{X_C} = \frac{200}{10^4} = \frac{2}{100} = 20\times 10^{-3}\,A = 20mA$$

14. In a pure idncutor circuit, $E = E_o \sin\omega t$

$$I_L = I_o \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{where } I_o = \frac{E_o}{X_L}$$

So the current lags behind the applied voltage by a phase angle of $\pi/2$ radian.

15. $R = 300\,\Omega$

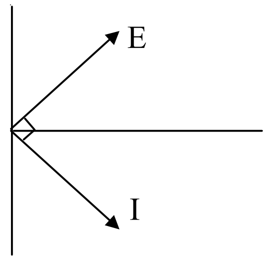
$X_L = \omega L = 100 \times 1 = 100\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\times 20\times 10^{-6}} = 500\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(300)^2 + (100-500)^2} = 500\,\Omega$$

16. $\tan\phi = \frac{X_L - X_C}{R} = \frac{X_L - 0}{0} = \infty \quad \Rightarrow \phi = 90^\circ$

Current lags behind the applied voltage by a phase angle of 90° or current leads the applied voltage by a phase angle of -90°.



17. Since, $V_L = V_C = V_R$

$\therefore \quad X_L = X_C = R$ and $V = V_R = 10\,V$

when capacitor is short circuited

$$i = \frac{10}{\sqrt{R^2 + X_L^2}} = \frac{10}{\sqrt{2}R} \quad \text{(as } X_{:L} = R)$$

$$V_L = iX_L = \left(\frac{10}{\sqrt{2}R}\right)R = \frac{10}{\sqrt{2}}V$$

18. $R = 100\Omega$

$X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 0.5 = 157\,\Omega$

$$X_C = \frac{1}{2\pi fc} = \frac{1}{314 \times 10 \times 10^{-6}} = \frac{10^5}{314}\Omega = 318.47\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100)^2 + (15.7 - 318.47)^2} = 189.72\ \Omega$$

19. In series R-L-C circuit, V_L and V_C are always oppsoite in phase.

$\therefore (\Delta V_c + \Delta V_L) = 7.4 - 2.6 = 4.8\,V$

20. $$\tan\phi = \frac{X_L}{R} = \frac{2\pi fL}{R} = \frac{2\pi(200/\pi)1}{200} = 2$$

$\therefore \quad \phi = 63^o$.

21. $L_{eq} = L + L = 2L$

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{C^2}{2C} = \frac{C}{2}$$

$$f = \frac{1}{2\pi\sqrt{L_{eq}C_{eq}}} = \frac{1}{2\pi\sqrt{2L \times \frac{C}{2}}} = \frac{1}{2\pi\sqrt{LC}}$$

22. $$\cos\phi = \frac{R}{|Z|} \text{ or } \frac{\sqrt{3}}{2} = \frac{\sqrt{300}}{|Z|} \text{ or } Z = 20\,\Omega$$

23. I = current flowing through condenser/capacitor.
R, L, C are connected in series. So same current flows through R.

$$V_R = IR \Rightarrow 15 = I \times 60 \Rightarrow I = \frac{1}{4} = 0.25\ A$$

24. $$\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$$

25. The DC ammeter reads average current which is zero for alternating current.

26. $V^2 = V_R^2 + V_L^2 \Rightarrow V_R = 3\,\text{volt}$

27. V_L leads the current where as V_C lags behind the current.

28. As $V_L = V_C = 300\,V$,

and $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$\therefore\ V_R = V = 220\,V$

Also $I = \frac{V}{R} = \frac{220}{100} = 2.2$ A.

29. $C_{eff} = C_1 + C_2 + C_3 = 4\,\mu F + 2.5\,\mu F + 3.5\,\mu F = 10\,\mu F$

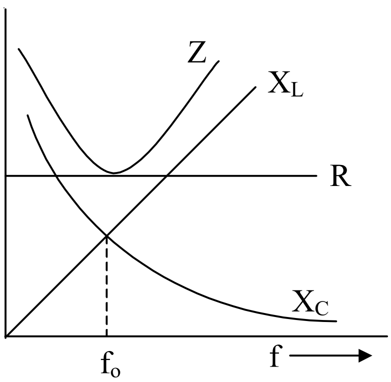
$L_{eff} = L_1 + L_2 = 1.7\,mH + 2.3\,mH = 4\,mH$

Resonance frequency, $\omega = \frac{1}{\sqrt{L_{eff}C_{eff}}} = \frac{1}{\sqrt{4\times10^{-3}\times10\times10^{-6}}} = \frac{10^4}{2} = 0.5\times10^4\,rad/s$

30. $\omega = \frac{1}{\sqrt{LC}}\,Q = \frac{L\omega}{R} = \frac{L}{R/\sqrt{LC}} = \frac{4\times10^{-3}}{100\sqrt{4\times10^{-3}\times40\times10^{-12}}} = 100$

31. $X_L = 2\pi fL \Rightarrow X_L \propto f$

$X_C = \frac{1}{2\pi fc} \Rightarrow X_c \propto \frac{1}{f}$



R is independent of frequency
f_o = resonance frequency.
$f < f_o$, $Z > R$
$f > f_o$, $Z = R$
$f > f_o$, $Z > R$
so shape of graph will be bel shaped.

32. $f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{2\pi}.\frac{1}{2\pi}\times10^{-6}}} = 1000\,Hz$

33. $f_o = \frac{1}{2\pi\sqrt{LC}}$

$f'_o = \frac{1}{2\pi\sqrt{L'C'}} = \frac{1}{2\pi\sqrt{L'2C}}$

As per the question $f'_o = f_o$

$\therefore\ L' = \frac{L}{2}$

34. $\omega = \frac{1}{\sqrt{LC}} Q = \frac{L\omega}{R} = \frac{L}{R/\sqrt{LC}} = \frac{4\times10^{-3}}{100\sqrt{4\times10^{-3}\times40\times10^{-12}}} = 100$

35. $E_{rms} = \frac{100}{\sqrt{2}}$ volt. $I_{rms} = \frac{100}{\sqrt{2}} mA = \frac{100}{\sqrt{2}}\times10^{-3}$ A, $\phi = \frac{\pi}{3}$

$P_{av} = E_{rms}I_{rms}\cos\phi = \frac{100}{\sqrt{2}}\times\frac{100}{\sqrt{2}}\times10^{-3}\times\cos\frac{\pi}{3} = \frac{100\times100}{2}\times10^{-3}\times\frac{1}{2} = 2.5\,\text{watt}$

36. $x = 4\,\Omega$, $R = 3\,\Omega$

$Z = \sqrt{R^2 + X^2} = \sqrt{3^2+4^2} = 5$

Power factor $= \cos\phi = \frac{R}{Z} = \frac{3}{5} = 0.6$

37. $P = \frac{V_P^2 R}{2Z^2} = \frac{282\times282\times500}{2\times\left(\sqrt{(500)^2+\left(\frac{10^6}{2\times377}\right)^2}\right)^2} = \frac{282\times282\times500}{2\times1410\times1410} = 10\,W$

38. As V = 5 cos (2π ft) =5 sin (2 π ft + $\pi/2$)

And I = 2 sin (2 πft)

$\therefore$ phase difference between V and I is $\phi = \frac{\pi}{2}$

Average power P = $V_{rms}I_{rms}\times\cos\phi = 0$

39. $P = E_{rms}I_{rms}\cos\phi = \frac{E_o}{\sqrt{2}}\times\frac{I_o}{\sqrt{2}}\cos\frac{\pi}{2} = 0$

40. $P = V_{rms}I_{rms}\cos\phi = \frac{100}{\sqrt{2}}.\left(\frac{100}{\sqrt{2}}\times10^{-3}\right)\cos60^o = \frac{10}{2}\times\frac{1}{2} = 2.5\,\text{watt}$.

41. $P = \frac{V_{rms}^2\times R}{|Z|^2} = \frac{50^2\times3}{5^2} = 300\,W$

LEVEL - II

42. $V = V_o \Rightarrow \sin(\omega t + \pi/3) = 1$ or $\omega t = \pi/6$

or $\frac{2\pi t}{T} = \pi/6$ or $t = \frac{T}{12}$ seconds

43. $\phi = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{2\times 2\pi f} = \frac{1}{8f}$

$\Rightarrow t = \frac{1}{8\times 50} = \frac{1}{400}$ sec = 2.5 ms

44. $V(t) = \left(\frac{4V_o}{T}\right)t$ for $0 \le t \le \frac{T}{4}$; $V^2 = \left(\frac{16V_o^2}{T^2}\right)t^2$

$\therefore \quad < V^2 >_{0-T/4} = \frac{\left(\frac{16V_o^2}{T^2}\right)\int_0^{T/4} t^2 dt}{\int_0^{T/4} dt} = \frac{V_o^2}{3}$

$\therefore \quad V_{rms} = \sqrt{\frac{V_o^2}{3}} = \frac{V_o}{\sqrt{3}}$

45. $e = e_1 \sin\omega t + e_2 \cos\omega t = \sqrt{e_1^2 + e_2^2}\left[\sin\omega t \frac{e_1}{\sqrt{e_1^2+e_2^2}} + \cos\omega t \frac{e_2}{\sqrt{e_1^2+e_2^2}}\right]$

$\Rightarrow \sqrt{e_1^2+e_2^2}\,[\sin\omega t\cos\phi + \cos\omega t\sin\phi]$

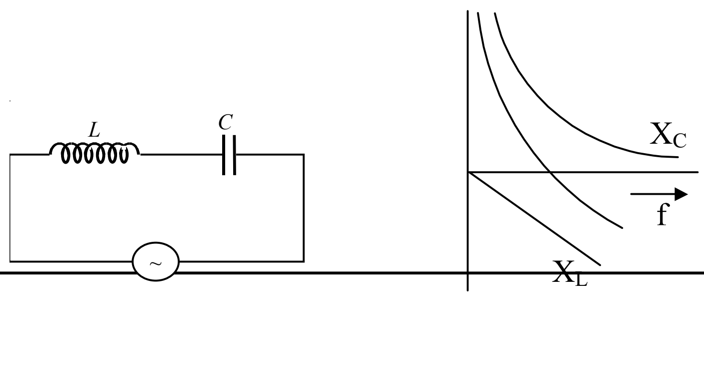
$\Rightarrow e = \sqrt{e_1^2+e_2^2}\,\sin(\omega t + \phi)$ where $\phi = \tan^{-1}\frac{e_2}{e_1}$

$e_{rms} = \frac{e_{max}}{\sqrt{2}} = \frac{\sqrt{e_1^2+e_2^2}}{\sqrt{2}} = \sqrt{\frac{e_1^2+e_2^2}{2}}$

46. $V_{rms} = \sqrt{\frac{1}{T}\int_0^T 10^2 dt} = 10V$

47. $i_{rms}\sqrt{\frac{1}{T}\int_0^T i^2 dt} = \frac{T^2}{\sqrt{5}}W$

48.



Assuming $X = X_C - X_L$

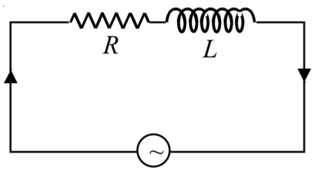
When t = 0, $X_C = \infty$, $X_L = 0$ $\Rightarrow$ $X = X_C - X_L = \infty$

when $f = f_o$, $X_C = X_L \Rightarrow X = 0$

when $f \to \infty$, $X_C \to 0, X_L \to -\infty$ $\Rightarrow$ $X \to -\infty$ so the graph will look like ograph (d).

49. $\omega = 2 \times 10^4$ rad/s

Let $E = E_o \sin \omega t$, $I = I_o \sin(\omega t - \phi)$



$$E = E_o \Rightarrow \omega t = \frac{\pi}{2}$$

$$I = \frac{I_o}{\sqrt{2}} \Rightarrow \omega t - \phi = \frac{\pi}{4}$$

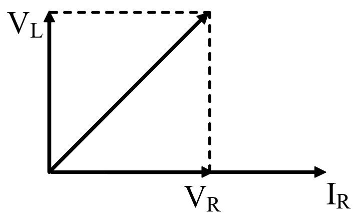
$$\therefore \phi = \frac{\pi}{4} \Rightarrow \tan\phi = 1$$

$$\Rightarrow \frac{\omega L}{R} = 1 \Rightarrow L = \frac{R}{\omega} = \frac{20}{2 \times 10^4} = 10^{-3}\,H = 1MH$$

50. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$\Rightarrow X_C \propto \frac{1}{f}$ $\Rightarrow$ Graph should be rectangular hyperbola.

51. $\tan\phi = \frac{4}{3} \Rightarrow \phi = 53^o$ $E = 10 \sin \omega t$



$I = \frac{10}{5} \sin(\omega t - 53)$ $I = 2\sin(\omega t - 53)$

$$V_L = IX_L \sin(\omega t - 53 + 90^o) = IX_L \sin(\omega t + 37)$$

At $t = \frac{\pi}{\omega}$, $V_L = 4.8\,V$

52. $V^2 = V_R^2 + V_C^2 \Rightarrow V_R = 6$ volt

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{V_C}{V_R}\right) = \tan^{-1}\left(\frac{8}{6}\right) = \tan^{-1}\left(\frac{4}{3}\right)$$

53. $I_{max} = \sqrt{2^2 + 2^2 + 2 \times 2 \times 2 \cos 30^o} = \sqrt{8 + 4\sqrt{3}}$

$$I_{rms} = \frac{1}{\sqrt{2}} I_{max} = \sqrt{\frac{8 + 4\sqrt{3}}{2}} = \sqrt{4 + 2\sqrt{3}}$$

54. $I_{rms}^2 = \frac{1}{0.01}\int_0^{0.01} I^2 dt = 4 \quad \Rightarrow I_{rms} = 2$

55. $P = VI\cos\phi = V \times \frac{V}{\sqrt{R^2+\omega^2L^2}} \times \frac{R}{\sqrt{R^2+\omega^2L^2}}$

$P = \frac{V^2R}{R^2+\omega^2L^2}$

56. Old power factor $= \frac{R}{Z} = \frac{R}{\sqrt{10}\,R} = \frac{1}{\sqrt{10}}$

New power factor $= \frac{R}{Z} = \frac{R}{\sqrt{5}\,R} = \frac{1}{\sqrt{5}}$

Ratio $= \sqrt{2}$

57. For maximum power to be transferred

$X_L = X_C$ or $L\omega = \frac{1}{C\omega}$

or $C = \frac{1}{L\omega^2} = \frac{1}{10\times(100\pi)^2} = 10^{-6}F$

58. $I = \frac{V}{\sqrt{R^2+(\omega L)^2}}$

Power loss=$I^2R = \frac{V^2R}{R^2+(\omega L)^2} = \frac{V^2}{R+\frac{\omega^2L^2}{R}}$

For minimum power loss, resistance should be low and inductance shold be high.

59. For pure inductor, $\phi = \frac{\pi}{2} \quad \Rightarrow \cos\phi = 0$

$P_{av} = E_v I_v \cos\phi = 0$

60. RMS current in choke coil = $I_{rms} = \frac{I_o}{\sqrt{2}}$

Power loss is due to resistance only in choke coil.

Powr loss = $I_{rms}^2 R = \frac{I_o^2}{2}R$

61. $Z = \frac{220}{2.2} = 100$, $R = Z$

$$X_C = X_L = 100\pi \times \frac{1}{\pi} = 100$$

$$\text{Powr factor of box} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{100}{\sqrt{100^2 + 100^2}} = \frac{1}{\sqrt{2}}$$

62. $P_{avg} = I^2R = 4 \times 5 = 20 \text{ watt}$

LEVEL - III

63. $$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times \sqrt{\frac{1}{\pi} \times \frac{1}{4\pi} \times 10^{-6}}} = 1000\,\text{Hz}$$

64. For L-R circuit, $E = E_o \sin\omega t$

$I = I_o \sin(\omega t - \phi)$ where $I_o = \frac{E_o}{Z}$

So current largs behind the applied voltage.

65. $Z = \sqrt{R^2 + \omega^2 L^2}$

$$\cos\phi = \frac{R}{2} \Rightarrow \frac{1}{2} = \frac{100}{2} \Rightarrow Z = 200\ \Omega$$

$$\therefore R^2 + \omega^2L^2 = Z^2 \quad \Rightarrow \omega^2L^2 = Z^2 - R^2$$

$$\Rightarrow \quad \omega^2L^2 = 4 \times 10^4 - (100)^2 = 3 \times 10^4 \Rightarrow \omega L = 100\sqrt{3}$$

$$\Rightarrow \quad 2 \times \pi \times 50 \times L = 100\sqrt{3} \Rightarrow L = \frac{100\sqrt{3}}{2\pi \times 50} = \frac{\sqrt{3}}{\pi}$$

66. $$I_{rms}^2 = \frac{1}{T}\int_0^T I^2 dt = \frac{1}{T}\int_0^T (I_o + I_1 \sin\omega t)^2\, dt$$

$$I_{rms}^2 = I_o^2 + 0.5\, I_1^2 \Rightarrow I_{rms} = \sqrt{I_o^2 + 0.5 I_1^2}$$

67. Case I: $I_1 = \frac{I_o}{\sqrt{2}}$

Case II: $I_2 = \frac{I_o}{\sqrt{2}}$

Case III: $I_3^2 = \frac{2}{T}\int_0^{T/2} I_o^2 dt = \frac{2}{T} I_o^2 \times \frac{T}{2} = I_o^2$

$I_3 = I_o$

Case IV: $$I_4^2 = \frac{2}{T}\int_0^{T/2} I^2 dt = \frac{2}{T}\int_0^{T/2} \frac{4I_o^2}{T^2}t^2 = \frac{8I_o^2}{T^3}\left[\frac{t^3}{3}\right]_0^{T/2}$$

$$I_4^2 = \frac{8I_o^2}{3T^3}\frac{T^3}{8} = \frac{I_o^2}{3} \Rightarrow I_4 = \frac{I_o}{\sqrt{3}}$$

68. Apply Lenz's law, current in both coils will be in same direction. So repulsion between two coils takes place.

69. $$\tan\phi = \frac{X_L - X_C}{R} = \frac{V_L - V_C}{V_R} = \frac{3-2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \quad \Rightarrow \phi = \frac{\pi}{6}$$

$X_L > X_C \Rightarrow$ Current lags the applied emf.

$$\therefore I = I_o \cos\left(\omega t' - \frac{\pi}{6}\right)$$

70. $$0.6 = \frac{R}{\sqrt{R^2 + X_L^2}} \quad \Rightarrow X_L = \frac{4}{3}R_1$$

$$0.5 = \frac{R}{\sqrt{R^2 + X_C^2}} \quad \Rightarrow X_C = \sqrt{3}\,R_2$$

Now in LCR circuit,

$$\cos\phi = \frac{R}{Z} \Rightarrow 1 = \frac{R_1 + R_2}{\sqrt{(R_1+R_2)^2 + (X_L - X_C)^2}}$$

$$\Rightarrow X_L = X_C \Rightarrow \frac{4}{3}R_1 = \sqrt{3}\,R_2 \Rightarrow \frac{R_1}{R_2} = \frac{3\sqrt{3}}{4}$$

MULTIPLE CORRECT ANSWERS TYPE
LEVEL - I

1. It is given that in series circuit, instantaneous current is zero when instantaneous voltage is maximum.

$\Rightarrow \quad \phi = 90° \Rightarrow R = 0$

$\Rightarrow$ circuit may be of pure capacitor, pure inductor or combination of inductor and capacitor.

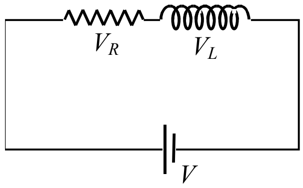
2. $V = 200\sin(100\pi t)$

Comparing the equation with $V = V_o \sin\omega t$ we have peak current

$V_o = 200\text{volt} \quad \Rightarrow \quad V_{rms} = \frac{200}{\sqrt{2}}\text{volt} = 100\sqrt{2}\text{ volt}$

$\omega = 100\pi \Rightarrow 2\pi f = 100\pi \Rightarrow f = 50\text{ Hz}$

3.



Let $I = I_o \sin\omega t$, $V_R = I_o R \sin\omega t$, $V_L = I_o X_L \sin\left(\omega t + \frac{\pi}{2}\right) = I_o X_L \cos\omega t$

$$I_{av} = \frac{1}{T}\int_0^T I dt = 0$$

$$V_{L_{av}} = \frac{1}{T}\int_0^T V_L dt = 0$$

Joule heat $\propto I^2$ so average value of joules heat $\neq 0$.

Energy stored in inductor = $\frac{1}{2}LI^2$, so energy $\neq 0$.

4. $E_S = \frac{N_S}{N_P} E_P$

Secondary emf does not depend upon the resistance of primary and secondary coil.

5. DC dynamo converts mechanical energy into DC voltage (Electrical energy).
AC dynamo converts mechanical energy into AC voltage (electrical energy).

6. P_{av} = $V_{rms} I_{rms} \cos\phi$

If $\cos\phi = 1$ then maximum average power = $V_{rms} I_{rms}$ = 1000 watt.

If cos $\phi < 1$ then average power will be less than 1000 wattt.

So correct option is b, d

7. At resonance frequency $X_L = X_C$. So the R-L-C series circuit behaves like resistive circuit. At frequency less than resonance frequency $X_C > X_L$. So circuit will behave as capcitive circuit.

8. Here $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times \frac{0.4}{\pi} = 40\Omega$

R = 30 Ω

$\therefore$ $Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = 50\Omega$

$I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{50} = 4A$.

9. $V^2 = V_R^2 + (V_L - V_C)^2$ But $V_L = V_C$

$\therefore$ $V_R = V = 200\,V$ and $i = \frac{V_R}{R} = 2\,A$

LEVEL - II

10. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Reactance of the circuit zero means Z = R only.

This is possible if both $X_L = 0$ and $X_C = 0$ or $X_L = X_C$

11. V_{rms} = 100 V. Peak value of voltage = $100\sqrt{2}V$. Peak value of current $= \frac{100\sqrt{2}}{50} = 2\sqrt{2}\,A$

$$I_{rms} = \frac{2\sqrt{2}}{\sqrt{2}} = 2A$$

12. Time constant of R-L circuit is L/R. So dimension of R/L will be that of frequency. Time constant of R-C circuit is RC.

$$\left[\frac{R}{L}\right] = \frac{1}{[L/R]} = \frac{1}{[t_C]} = \left[M^0L^0T^{-1}\right]$$

$$\left[\frac{1}{RC}\right] = \frac{1}{[RC]} = \frac{1}{[t_C]} = \left[M^0L^0T^{-1}\right]$$

$$\left[\frac{1}{\sqrt{LC}}\right] = [\omega_o] = [2\pi f] = \left[M^0L^0T^{-1}\right]$$

13. In series LCR circuit current remains same.

Let $I = I_o \sin\omega t$

$V_R = I_o R \sin\omega t$

$$V_L = I_o X_L \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$V_C = I_o X_C \sin\left(\omega t - \frac{\pi}{2}\right)$$

$\therefore$ V_L leads the current by a phase angle of $\frac{\pi}{2}$ radian.

V_C lags behind the current by phase anlge of $\frac{\pi}{2}$ radian.

V_R is in phase with I.

But voltage across series combination of LCR is V = $\sqrt{V_R^2 + (V_L - V_C)^2}$

14. At resonance $X_L = X_C$ and Z = R (minimum)

$$\tan\phi = \frac{X_L - X_C}{R} = 0 \quad \Rightarrow \phi = 0$$

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{E}{R} \text{ (maximum).}$$

So current is maximum and in phase with applied voltage.

15. As $\frac{dv}{dt} = \text{constant}$ and for capacitor $q = CV \Rightarrow \frac{dq}{dt} = C\frac{dV}{dt}$

$I =$ constant

16. $R = 11\Omega,\ X_L = X_C = 120\Omega$

$$Z = 11\Omega, I = \frac{110}{11} = 10\,A$$

$$V_C = V_L = 120 \times 10 = 1200\,V$$

$$V_R = 110\,V$$

17. $I = \frac{V}{X_L}$

$$\frac{X_{L_1}}{X_{L_2}} = \frac{2\pi f_1 L}{2\pi f_2 L} = \frac{f_1}{f_2} = \frac{1}{8} \qquad \frac{I_1}{I_2} = \frac{X_{L_2}}{X_{L_1}} = 8 \Rightarrow I_2 = \frac{1}{8} I_1$$

LEVEL - III

18. At resonance $X_L = X_C$ so $Z = R \Rightarrow I = \frac{V}{R}$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi} \times \frac{1}{\pi} \times 10^{-6}}} = 500\,Hz$$

In series RLC circuit, phase angle between V_L and V_C is 180°.

19. $Z = \sqrt{R^2 + X_C^2} = \sqrt{(300)^2 + (400)^2} = 500,\ X_C = \frac{1}{\omega C} = 400\,\Omega$

$$I_o = \frac{V_o}{Z} = 0.1 \text{ amp}$$

$$P_{av} = E_{rms} I_{rms} \cos\phi = \frac{50}{\sqrt{2}} \times \frac{0.1}{\sqrt{2}} \times \frac{3}{5} = 1.5 \text{ watt}$$

20. $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$; $I_{rms} = \frac{V_{rms}}{Z}$; $V_C = I_{rms} X_C$

21. $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$; $P_{av} = E_{rms} I_{rms} \cos\phi$, $\cos\phi = \frac{R}{Z}$

22. $I_o = \frac{E_o}{X_L} = \frac{10}{\omega L}$

Now put the data given in different options to find out the correct answers.

23. $X_L = \omega L = 2\pi f L = 2.\frac{22}{7} \times 70 \times 1 = 440\,\Omega$; $I_{rms} = \frac{V}{X_L} = \frac{110}{440} = 0.5\text{ Amp}$

COMPHREHENSIVE TYPE QUESTIONS

1. $i = \frac{V_R}{R} = \frac{(2.0\text{V})\sin(10^3 t)}{100} = (2.0 \times 10^{-2}\text{ A})\sin(10^3 t)$

2. $X_L = \omega L = (10^3) \times (4\text{H}) = 4.0 \times 10^3 \text{ ohm}$

3. Amplitude of voltage across inductor, $V_0 = I_0 X_L = (2.0 \times 10^{-2}\text{ A})(4.0 \times 10^3 \text{ ohm})$ = 80 volts

4. rms value of voltage across the source $V_{rms} = \frac{100\sqrt{2}}{\sqrt{2}} = 100\text{ Volt}$

From question, $\omega = 1000\text{ rad/s}$ $i_{rms} = \frac{V_{rms}}{|Z|} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{rms}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

$$= \frac{100}{\sqrt{(1000)^2 + \left(1000 \times 2 - \frac{1}{1000 \times 1 \times 10^{-6}}\right)^2}} = 0.0707\text{ amp}$$

Since the curernt will be same every where in the circuit, therefore

P.D. across resistor $V_R = i_{rms} R = 0.0707 \times 1000 = 70.7\text{ volts}$

5. P.D. across inductor $V_L = i_{rms} XL = 0.0707 \times 1000 \times 2 = 141.4\text{ volt}$

6. P.D. across capalitor $V_C = i_{rms} X_C = 0.0707 \times \frac{1}{1 \times 1000 \times 10^{-6}} = 70.7\text{ volts}$

7. $V_{out} = \frac{V_S}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \times \sqrt{R^2 + \omega^2 L^2}$; $\frac{V_{out}}{V_s} = \frac{\sqrt{R^2 + \omega^2 L^2}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

8. As $\omega \to 0, \quad \frac{V_{out}}{V_S} = \omega C R$

9. As $\omega \to \infty, \frac{V_{out}}{V_S} = 1$

10. AC ammeters are based on heating effect of current. Heat produced depends on RMS current. Hence AC measuring instrument measures its rms value.

11. (a) $V_{rms} = \sqrt{\frac{2}{T}\int_0^{T/2} V^2 dt}$ $\quad V_{rms}^2 = \frac{2}{T}\int_0^{T/2} V_o^2 dt = \frac{2}{T}.V_o^2 \times \frac{T}{2}$ $\quad V_{rms} = V_o$

(b) $V_{rms} = \sqrt{\frac{2}{T}\int_0^{T/2} V^2 dt} = \frac{2V_o}{\sqrt{3}}$ (c) $V_{rms} = \frac{2V_o}{\sqrt{2}} = \sqrt{2}\,V_o$ (d) $V_{rms} \neq V_o$ So correct option is (a).

12. $V_{av} = \frac{\text{Area of the graph for time } t = 0 \text{ to } T}{\text{Time interval}}$ $\quad V_{av} = \frac{\frac{1}{2} \times \frac{T}{2} \times 2V_o + 0}{T} = \frac{V_o T}{2T} = \frac{V_o}{2}$

MATRIX MATCHING TYPE QUESTIONS

1. (A) R-LC circuit at resonance

At resonance, $X_L = X_C \Rightarrow$ Net reactance = 0.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

If $E = E_o \sin \omega_o t$ then $I = I_o \sin \omega_o t$ where $I_o = \frac{E_o}{R}$

$\Rightarrow$ Frequency of current is saem as frequency of alternating voltage source.

$$\tan\phi = \frac{X_L - X_C}{R} = 0 \Rightarrow \phi = 0$$

$\Rightarrow$ current is in phase with applied voltage.

(B) Only resistor in an ac circuit

Frequency of current is same as frequency of applied voltage.

Reactance of the circuit is zero.

$$\tan\phi = \frac{X}{R} = 0 \Rightarrow \phi = 0$$

So current is in phase with applied voltage.

(C) Only inductor in ac circuit

Let applied voltage $E = E_o \sin \omega t$

Current in the circuit, $I = I_o \sin\left(\omega t - \frac{\pi}{2}\right)$, where $I_o = \frac{E_o}{X_L}$

$\therefore$ Frequency of current is same as applied voltage.

Current lags behind applied voltage.

(D) Only capacitor in an a.c. circuit

Current $I = I_o \sin\left(\omega t + \frac{\pi}{2}\right)$

Frequency of current is same as applied voltage.

Current leads the applied voltage by $\frac{\pi}{2}$.

2. (A) Square wave having peak value V_o.

$$V_{av} = \frac{2}{T}\int_0^{T/2} Vdt = V_o$$

$$V_{rms}^2 = \frac{2}{T}\int_0^{T/2} V^2 dt \Rightarrow \quad V_{rms} = V_o$$

$$V_o = V_{rms} = V_{av}$$

(B) Sinusoidal wave: $V = V_o \sin\omega t$

$$V_{av} = \frac{2V_o}{\pi},\ V_{rms} = \frac{V_o}{\sqrt{2}}$$

$$\therefore\ V_o > V_{rms} > V_{av}$$

(C) In pure capacitor circuit current leads the voltage by $\frac{\pi}{2}$.

(D) Wattless current flows due to reactance of the circuit.
(i.e. X_L, X_C or both)

3. (A) For ω = $8000\,\text{rad/sec}$

$$X_L = \omega L = 8000 \times 10 \times 10^{-3} = 80\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{8000 \times 10^{-6}} = 125\,\Omega,\quad R = 100\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100)^2 + (80-125)^2} = 109.6$$

$$I_o = \frac{V_o}{Z} = \frac{10}{109.6}\text{Amp} < 0.1\text{Amp}$$

As $X_C > X_L$ so current through the circuit wll lead the voltage across it.

(B) $R = 100\Omega$

$$\omega = 10000\,\text{rad/s} = 10^4\,\text{rad/s}$$

$$X_L = \omega L = 10^4 \times 10 \times 10^{-3} = 10^2\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{10^4 \times 10^{-6}} = 10^2\ \Omega$$

$X_L = X_C \Rightarrow$ circuit is in resonance. $[\phi = 0]$.

So voltage across the combination and the current will be in same phase.

$$I = \frac{V_o}{Z} = \frac{10}{100} = 0.1\text{ Amp}$$

(C) $\omega = 10,500\,\text{rad/s}$

$\omega_o = 10000\,rad/s$

$\omega > \omega_o \quad \Rightarrow X_L > X_C \quad \Rightarrow$ Voltage across the combination leads the current.

$Z = \sqrt{R^2 + (X_L - X_C)^2} > R$

$\Rightarrow$ current will be less than 0.1 Amp..

[Maximum current is $I_{max} = \frac{V_o}{R} = 0.1$ Amp]

(D) $\omega = 1000\,rad/s = 10^3\,rad/s$

$R = 50\,\Omega$

$X_L = \omega L = 10^3 \times 10 \times 10^{-3} = 10\,\Omega$

$X_C = \frac{1}{\omega L} = \frac{1}{10^3 \times 10^{-6}} = 10^3\ \Omega$

$X_C > X_L \Rightarrow$ current through the combination leads the applied voltage.

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50)^2 + (990)^2}$

$\Rightarrow I_o = \frac{V_o}{Z} = \frac{10}{\sqrt{(50)^2 + (990)^2}} < 0.1$ Amp

4. $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

(a) At resonance, $X_L = X_C$. So Z = R = 40Ω

(b) At resonance Z = R = 40Ω

$I_{rms} = \frac{200}{40} = 5\,A \Rightarrow I_o = 5\sqrt{2}\,A$

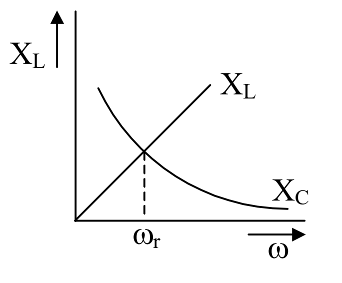
(c) $\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50$

$X_L = \omega_o L = 50 \times 5 = 250\,\Omega$

$V_L = I_{rms} X_L = 250 \times 5 = 1250$ volt

(d) At resonance, $V_R = IR = 5 \times 40 = 200$ volt

5.



(A) If $\omega > \omega_r$ then $X_L > X_C$, the circuit will be inductive in nature. So voltage will lead the current.

(B) If $\omega = \omega_r$ and $X_L = X_C$, Z = R.

The circuit will be resistive in nature, so the current will be in phase with voltage. $(B \to S)$

(C) If $\omega = 2\omega_r$ then $X_L > X_C$. So voltage will lead the current.

When $\omega = \omega_r$, $X_L = X_C$.

If $\omega = 2\omega_r$ then $X'_L = (2\omega_r)L = 2\omega_r L = 2X_L = 2X_C$.

So if $\omega = 2\omega_r$ then new inductive reactance will be two times the capacitive reactance in magnitude.

(D) If $\omega < \omega_r$ then $X_L < X_C$. The circuit will be capacitive in nature.
So the current will lead the applied voltage.

6. $\tan\phi = \dfrac{X_L - X_C}{R}$, $\omega = 200\,\text{rad/s}$

(A) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{200 \times 500 \times 10^{-6}} = \dfrac{10^6}{10^5} = 10\,\Omega$

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{0-10}{10} = -1$$

$\phi = -\dfrac{\pi}{4} \Rightarrow$ magnitude of phase difference = $\dfrac{\pi}{4}$

Current leads the applied voltage.

(B) $X_L = \omega L = 200 \times 5 = 1000\,\Omega$

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{1000-0}{0} = \infty$$

$\Rightarrow \phi = \dfrac{\pi}{2}$. So the current lags behind the applied voltage.

(C) R = 0

$$\tan\phi = \frac{X_L - X_C}{R} = \infty \Rightarrow \phi = \frac{\pi}{2}$$

$X_L = 200 \times 4 = 800\ \Omega$

$$X_C = \frac{1}{200 \times 3 \times 10^{-6}} = \frac{10^6}{6 \times 10^2} = \frac{1000}{6} = 166.66$$

$X_L > X_C \Rightarrow$ The current lags in phase to source voltage.

(D) $R = 1000\,\Omega$, $X_L = 200 \times 5 = 1000\,\Omega$

$$\tan\phi = \frac{X_L - X_C}{R} = 1 \Rightarrow \phi = \frac{\pi}{4}$$

Current lags in phase to source voltage.

7. $I_{rms} = \sqrt{\frac{1}{T}\int_0^T I^2 dt}$

In half cycle, $I_{av} = \frac{2}{T}\int_0^{T/2} I dt$

In full cycle, $I_{av} = \frac{1}{T}\int_0^T I dt$

Now calculate for different wave form.

(A) $I_{rms} = \frac{I_o}{\sqrt{2}}$

In half cycle, $I_{av} = \frac{2I_o}{\pi}$ In full cycle, $I_{av} = 0$.

(B) $I = \frac{4I_o}{T}t \quad 0 < t < \frac{T}{4}$

$$I_{rms}^2 = \frac{4}{T}\int_0^{T/4} I^2 dt = \frac{4}{T}\int_0^{T/4}\left(\frac{4I_o}{T}t\right)^2 dt = \frac{I_o}{3}$$

$I_{rms} = \frac{I_o}{\sqrt{3}}$ In half cycle, $I_{av} = \frac{\text{area}}{\text{time}} = \frac{\frac{1}{2}\times\frac{T}{2}\times I_o}{\frac{T}{2}} = \frac{I_o}{2}$

In full cycle $I_{av} = 0$

(C) $I = I_o$ 0 < t < T/2

$$I_{rms}^2 = \frac{2}{T}\int_0^{T/2} I^2 dt = \frac{2}{T}.I_o^2 \times \frac{T}{2} = I_o^2$$

$I_{rms} = I_o$

In half cycle, $I_{av} = \frac{\text{area}}{\text{time}} = \frac{\frac{T}{2}\times I_o}{\frac{T}{2}} = I_o$

In full cycle $I_{av} = 0$

(D) $I = I_o$ 0 < t < T/2

I = 0 T/2 < t < T

$$I_{rms}^2 = \frac{1}{T}\int_0^T I^2 dt = \frac{1}{T}\left[\int_0^{T/2} I_o^2 dt + \int_{T/2}^T 0 dt\right] = \frac{1}{T}I_o^2\frac{T}{2} = \frac{I_o^2}{2}$$

$\Rightarrow I_{rms} = \frac{I_o}{\sqrt{2}}$ In half cycle, $I_{av} = I_o$. In full cycle, $I_{av} = I_o/2$

ASSERTION REASONING TYPE QUESTIONS

1. Definition of rms current,

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}}$$

If $I = I_o \sin\omega t$ then $I_{rms} = \sqrt{\frac{1}{T}\int_0^T I^2 dt}$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_o^2 \sin\omega t dt} = \frac{I_o}{\sqrt{2}}$$

So both statements are true but statement-2 is not correct explanation of statement-1.

2. Average value of current in the positive half cycle cancels out the average value of current in the negative half cycle. So average value in one full cycle = 0.

$I_{av} = \frac{2I_o}{\pi}$ in the positive half cycle. $I_{av} = -\frac{2I_o}{\pi}$ in negative half cycle.

Statement 1 is true and statement 2 is false.

3. In a series R-L-C circuit resonance can takes place, because at a particular frequency of ac source, the impedance can be minimum.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Resonance means current is maximum $\Rightarrow$ Impedance is minimum $\Rightarrow X_L = X_C$.

4. AC ammeters are based on heating effect of current.

Heat produced = H = I^2Rt $\Rightarrow H \propto I^2 \Rightarrow I \propto \sqrt{H}$

So linear scale is not possible.

$\therefore$ Divsions in the AC ammeter are not equally placed.

5. In series R-L-C circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance I should be maximum $\Rightarrow$ Impedance Z should be minimum $\Rightarrow X_L = X_C$

and the minimum impedance is R (at resonance)

6. $Z=\sqrt{R^2+\left(X_L-X_C\right)^2}$

At resonance $X_L=X_C \Rightarrow \omega_o L=\frac{1}{\omega_o C} \Rightarrow 2\pi f_o L=\frac{1}{2\pi f_o C} \Rightarrow f_o=\frac{1}{2\pi\sqrt{LC}}$

Resonance frequency depends on L and C.

INTEGER TYPE QUESTIONS

1. When the capacitance is removed, the circuit becomes LR with

$$\tan\phi=\frac{X_L}{R}, \text{ i.e. } X_L=R\tan\phi=10\sqrt{3}$$

and when inductance is removed, the circuit becomes CR with

$$\tan\phi=\frac{X_C}{R}, \text{ i.e. } X_C=R\tan\phi=10\sqrt{3}\,\Omega$$

as here $X_L = X_C$, so the circuit is series resonance and hence as

$$X=X_L-X_C=0, \text{ i.e. } Z = R, \text{ so } \quad I_{rms}=\frac{V_{rms}}{Z}=\frac{V_{rms}}{R}=\frac{200}{10}=20\,A$$

and $P_{av}=V_{rms}I_{rms}\cos\phi=200\times20\times1=4\,kW$

2. $$V_{av}=\frac{1}{2}\left[\int_0^1 v_1 dt+\int_1^2 v_2 dt\right]=\frac{1}{2}\left[\int_0^1 4dt+\int_1^2\left(-4t+4\right)dt\right]=\frac{1}{2}\left[\left|4t\right|_0^1+\left|-\frac{4r^2}{2}+4t\right|_1^2\right]=1\text{volt}$$

3. In case of direct current, $P_{DC}=\frac{V_M^2}{R}$

The impedance of LR circuit is

$$Z=\sqrt{R^2+X_L^2}=\sqrt{R^2+\omega^2L^2}$$

$$\therefore \quad I_M=\frac{V_M}{Z}=\frac{V_M}{\sqrt{R^2+\omega^2L^2}}$$

$$\therefore \quad P_{AC}=I_M^2R=\frac{V_M^2R}{R^2+\omega^2L^2}$$

According to the problem,

$$P_{DC}=\eta P_{AC} \quad \text{or} \quad \frac{V_M^2}{R}=\eta\frac{V_M^2R}{R^2\left\{1+\left(\frac{\omega L}{R}\right)^2\right\}}$$

or $$\frac{1}{R}=\frac{\eta}{R\left\{1+\left(\frac{\omega L}{R}\right)^2\right\}}$$ or $$\omega=\frac{R}{L}\sqrt{\eta^2-1} \Rightarrow 2\pi v=\frac{R}{L}\sqrt{\eta^2-1}$$

$$v=\frac{R}{2\pi L}\sqrt{\eta^2-1}=2\,kHz$$

4. At resonance reactance X = 0

$$i=\frac{V}{R}=\frac{60}{120}=\frac{1}{2}A$$

As $V_L = iX_L = i\omega L$ or $L=\frac{V_L}{i\omega}=\frac{40}{\left(\frac{1}{2}\right)\times 4\times 10^5}$

L = 0.2 mH

At resonance $\omega_o=\frac{1}{\sqrt{LC}}$ or $C=\frac{1}{\omega_o^2 L}=\frac{1}{0.2\times 10^{-3}\times\left(4\times 10^5\right)^2}=\frac{1}{32}\mu F$

In LCR circuit, $\tan\phi=\frac{X_L-X_C}{R}$

$$\Rightarrow 1\times 120=\omega\times 2\times 10^{-4}-\frac{1}{\omega\left(\frac{1}{2}\right)\times 10^{-6}} \quad\Rightarrow\quad \omega^2-6\times 10^5\,\omega-16\times 10^{10}=0$$

$$\Rightarrow \qquad \omega=\frac{6\times 10^5\pm\sqrt{\left(6\times 10^5\right)^2+64\times 10^{10}}}{2}=8\times 10^5\ rad/s$$

5. $\omega_o=\frac{1}{LC}=\frac{1}{\sqrt{10^{-2}\times 10^{-6}}}=10^4\ rad/s$

$w=w_o-\frac{10}{100}\omega_o=9\times 10^3$ rad/s

Hence $X_L=\omega L=9\times 10^3\times 10^{-2}=90\,\Omega$

$$X_C=\frac{1}{\omega C}=\frac{1}{9\times 10^3\times 10^{-6}}=111.11\Omega$$

$$X=X_C+X_L=111.11-90=21.11\Omega$$

Impedance $Z=\sqrt{R^2+X^2}=\sqrt{3^2+\left(21.11\right)^2}$

$$Z=21.32\,\Omega$$

$$I_o=\frac{15}{21.32}=0.704\,A$$

$$P_{av}=i_{rms}^2 R=\frac{i_o^2}{2}R=\frac{1}{2}\times\left(0.704\right)^2\times 3=0.74\ watt$$

$$f=\frac{W}{2\pi}=\frac{9\times 10^3}{2\pi}\ cycle/s$$

So energy per cycle $=\frac{0.74}{\frac{9\times 10^3}{2\pi}}=5.16\times 10^{-4}\approx 5\times 10^{-4}\ Joules/cycle$

6. When frequency of supply is equal to natural frequency,

then $X_L = X_C$. $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

$$P_{av} = VI\cos\phi = V.\frac{V}{R}\times 1 = \frac{V^2}{R} = \frac{200\times 200}{20}$$

$$P_{av} = 2000\,\text{watt} = 2\text{kW}$$

7. Power factor = $\cos\phi = 1 \Rightarrow X_L = X_C$

$$2\pi fL = \frac{1}{2\pi fc} \Rightarrow L = \frac{1}{4\pi^2 f^2 c} \quad \Rightarrow L = \frac{1}{4\times(3.14)^2\times(50)^2\times 5\times 10^{-6}} = 2H$$

8. To find the impedance of the circuit, we first calculate X_L and X_C.

$$X_L = 2\pi fL$$

$$= 2\times 3.14\times 50\times 25.48\times 10^{-3}\,\Omega = 8\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 796\times 10^{-6}} = 4\Omega$$

Therefore, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8-4)^2} = 5\Omega$

SUBJECTIVE QUESTIONS

1. (a) for v = v_0 sin ωt

$$I = \frac{v_0}{\left[\omega L - \frac{1}{\omega C}\right]}\sin\left[\omega t + \frac{\pi}{2}\right] \text{ if } R = 0$$

$$I_0 = 11.6\text{A}, I_{rms} = 8.24\text{A}$$

(b) $V_{L\,rms} = 207\text{V}, V_{C\,rms} = 437\text{V}$

(c) Whatever be the current I is L, actual voltage loads current by $\frac{\pi}{2}$ therefore average power consumed by 2 is zero.

(d) For C, Voltage lags by $\frac{\pi}{2}$. Again average power consumed by C is zero.

(e) Total average power absorbed is zero.

2.(a) Amplitude of current in maximum at resonance frequency.

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{0.12\times 480\times 10^{-9}}} = 663 \text{ Hz}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{230}{23} = 10\text{A}$$

$$I_o = I_{rms}.\sqrt{2} = 14.14\text{ A}$$

(b) Resonance frequency, power absorbed by circuit is maximum.
So f = 663 Hz.

$$P_{av} = I^2R = (10)^2 \times 23 = 2300 \text{ watt}$$

(c) Q factor = $\frac{\omega_r}{2\Delta\omega} = \frac{\omega_r L}{T} \Rightarrow \Delta\omega = \frac{R}{2L}$

$$\Rightarrow \Delta\omega = \frac{23}{2 \times 0.12} = \frac{23}{0.24} = 95.833$$

Let f be half power frequency.

$$2\pi(f \sim f_o) = 95.833 \Rightarrow f \sim f_o = 15 \text{ Hz}$$

$$f = f_o \pm 15 = 663 \pm 15 = 648 \text{ Hz and } 678 \text{ Hz}$$

$$I^2R = \frac{1}{2}P_{max} = \frac{1}{2}I_o^2R \Rightarrow I = \frac{I_o}{\sqrt{2}} = 10 \text{ Amp}$$

(d) Q factor = $\frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{23}\sqrt{\frac{0.12}{480 \times 10^{-9}}} = 21.7$

3. $Z = \sqrt{R^2 + X_C^2}$, $I = \frac{V}{\sqrt{R^2 + X_C^2}}$

$$I' = \frac{V}{\sqrt{R^2 + (3X_C^2)}} \qquad \frac{I'}{I} = \frac{\sqrt{R^2 + X_C^2}}{\sqrt{R^2 + 9X_C^2}} \qquad \frac{1}{4} = \frac{R^2 + X_C^2}{R^2 + 9X_C^2}$$

$$R^2 + 9X_C^2 = 4R^2 + 4X_C^2$$

$$5X_C^2 = 3R^2$$

$$\frac{X_C}{R} = \sqrt{\frac{3}{5}}$$

4. $R = 20\Omega$

$$X_L = 2\pi fL = 2\pi \times 50 \times 100 \times 10^{-3} = 10\pi = 31.4\,\Omega$$

$$X_C = \frac{1}{2\pi fc} = \frac{1}{100\pi \times 30 \times 10^{-6}} = \frac{10^6}{3\pi \times 1000} = \frac{10^3}{3\pi} = \frac{1000}{3\pi}$$

$X_C > X_L$ so current will lead the applied emf.

$$\tan\phi = \frac{X_L - X_C}{R} \Rightarrow \phi = 75^\circ \text{ leading.}$$

5. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$I_{rms} = \frac{V}{2}$$

Impedance of coil = $Z_1 = \sqrt{R^2 + X_L^2}$

Potential drop across the coil = $I_{rms}Z_1 = I_{rms}\sqrt{R^2 + X_L^2}$

Potential drop across capacitor = $V_C = I_{rms}X_C$

6. $V_{rms} = 500 \text{ volt}$, $f = 120 \text{ cycle/sec}$, $R = 4\Omega$, L = 0.68 H

$Z = \sqrt{R^2 + X_L^2}$, Effective current = $I_{rms} = \frac{V_{rms}}{Z}$

Power factor = $\cos\phi = \frac{R}{Z}$

7. Total resistance $R = 50 + 2 = 52\,\Omega$

L = 0.3 H, $C = 40 \times 10^{-6}$ F

$Z = \sqrt{R^2 + (X_L - X_C)^2}$; $X_L = \omega L = 2 \times \pi \times 50 \times 0.3 = 30\pi$

$X_C = \frac{1}{\omega C} = \frac{1}{2 \times \pi \times 50 \times 40 \times 10^{-6}} = 250\,\pi$

$X_C > X_L$ so current leads the applied voltage.

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(52)^2 + (30\pi - 250\pi)^2}$

$I_{rms} = \frac{V_{rms}}{2}$

$P_{av} = E_{rms}I_{rms}\cos\phi = E_{rms}\frac{E_{rms}}{2} \times \frac{R}{2} = \frac{E_{rms}^2 R}{2}$

8. (a) $E = \frac{1}{2}\frac{q^2}{C} = 1$ (b) $w = \frac{1}{\sqrt{LC}}$ 10^3 rad/sec

(c) $q = q_0 \cos\omega t$

(i) energy stoed completely electrical at $t = 0, \frac{T}{2}, T, \frac{3T}{2}, ...$

(ii) energy stored completely magnetic (i.e. electrical energy zero)

at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, ...$ where $T = \frac{1}{v} = 6.3$ ms

(d) At $T = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, ...$ because $q = q_0 \cos\frac{\omega T}{8} = q_0 \cos\frac{\pi}{4} = \frac{q_0}{\sqrt{2}}$

(e) R damps out the LC oscillations eventually. The whole of the initial energy (= 1.0 J) is eventually dissipated as heat.

9. For lamp, i = 5/20 = 0.25 A, R =20/0.25 = 80 Ω
Curent through the lamp should be 0.25 A.
(i) When condenser C is placed in series

$$i = \frac{200}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = 0.25$$

Putting the value of $\omega = 2\pi \times 50,$ we get $C = 4.0\,\mu F$

(ii) When inductor is used, $I = \frac{200}{\sqrt{R^2 + (\omega L)^2}} = 0.25 \Rightarrow L = 2.53\,H$

(iii) When resistance is used, $I = \frac{200}{R + r} = 0.25 \Rightarrow r = 720\,\Omega$

10. For a coil, $Z = \sqrt{R^2 + \omega^2 L^2},\ I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$

For dc source, $\omega = 0$, $I = \frac{V}{R}$, i.e. $R = \frac{15}{5} = 3\,\Omega$ (1)

When ac is applied,

$$I = \frac{V}{Z} \quad \text{i.e.} \quad Z = \frac{15}{3.0} = 5\,\Omega$$

$$\therefore \quad R^2 + X_L^2 = 25$$

$$X_L^2 = 25 - 9 = 16\,\Omega \Rightarrow X_L = 4\,\Omega$$

$$L = \frac{4}{40} = 0.08\,\text{Henry}$$

Now, when the capacitor is connected in series,

$$\therefore \quad X_C = \frac{1}{\omega C} = \frac{1}{50 \times 2500 \times 10^{-6}} = 8\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3 + (4-8)^2} = 5\,\Omega$$

$$\therefore \quad I = \frac{15}{5} = 3\,A$$

$$\therefore \quad P_{av} = V_{rms} I_{rms} \cos\phi = I_{rms}^2 R = (3)^2 \times 3 = 27\,W$$

Resonance frequency,

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.08) \times 2500 \times 10^{-6}}} = \frac{1000}{2\pi \times 5 \times 2\sqrt{2}} = \frac{25\sqrt{2}}{\pi} = 11.25\,Hz$$

11. $\omega_1 L = 250\,\Omega \quad \Rightarrow \omega_2 L = 2 \times 250 = 500\,\Omega$

$$\frac{1}{\omega_1 C} = 400\,\Omega \ \& \ \frac{1}{\omega_2 C} = 200\,\Omega\ ; \quad R = 400\,\Omega\ ; \quad \therefore \quad Z = \sqrt{(X_L - X_C)^2 + R^2} = 500\,\Omega$$

$$\therefore \quad \cos\phi = \frac{R}{Z} = 0.8$$

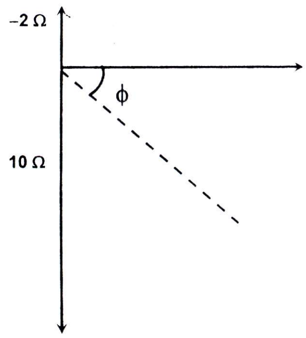
12. $C' = 0.1F.$ capacitor are in parallel.

$$\frac{1}{\omega C'} = \frac{1}{50 \times 0.1} = 0.2\Omega$$

$$\omega L' = 50 \times 0.2 = 10\Omega \quad \Rightarrow \quad \cos\phi = \frac{1}{\sqrt{2}} \Rightarrow \phi = 45°$$

$$\therefore \quad R = 10\Omega - 0.2\Omega = 9.8\Omega \quad \Rightarrow Z = \sqrt{9.8^2 + 9.8^2} = 9.8\sqrt{2}\ \Omega$$

$$P_{av} = \frac{V_{rms}^2}{Z}\cos\phi = \frac{200 \times 200}{500} \times 0.8 = 64\,W$$



$\therefore$ Peak current = $\frac{100}{9.8 \times \sqrt{2}} = 7.22$ Amperes.

13, $X_L = 2\pi f L = 2\pi \times \frac{300}{2\pi} \times 1 = 300\Omega$; $R = 300\Omega$; $\tan\phi = \frac{X_L}{R} = 1 \Rightarrow \phi = \frac{\pi}{4}$ (lagging)

14. Frequency of oscillation of an LC circuit is $v = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$ or $C = \frac{1}{4\pi^2 v^2 L}$.

where $L = 400\mu H = 400 \times 10^{-6} H$. For v = 500 kHz = 500 × 10^3 Hz, we have

$$C = \frac{1}{4\pi^2 \times (500 \times 10^3)^2 \times 400 \times 10^6} = 250 \times 10^{-12} F = 250\,pF$$

Similarly for v = 1 MHz = 10^6 Hz, we get C = 2.5 pF. Hence the range of the capacitor is 2.5 pF to 250 pF.

15. Resistance of bulb = R = $\frac{V^2}{P} = \frac{110 \times 110}{55} = 220\Omega$

$$\frac{VR}{\sqrt{R^2 + \omega^2 L^2}} = 110 \Rightarrow \frac{220 \times R}{\sqrt{R^2 + \omega^2 L^2}} = 110 \quad \Rightarrow 4R^2 = R^2 + \omega^2 L^2 \Rightarrow \omega L = \sqrt{3}R$$

$$\Rightarrow L = \frac{\sqrt{3}R}{\omega} = \frac{\sqrt{3} \times 220}{2\pi \times 50} = 1.2H$$

16. As the current lags behind the potential difference, the circuit contains resistance and inductance.

Power, P = $v_{rms} \times i_{rms} \times \cos\phi$; Here, $i_{rms} = \frac{V_{rms}}{Z}$, where $Z = \sqrt{[(R^2 + (\omega L)^2)]}$

$\therefore \quad P = \frac{V_{rms}^2 \times \cos\phi}{Z} \text{ or } Z = \frac{v_{rms}^2 \times \cos\phi}{P}$; So, $Z = \frac{(220)^2 \times 0.8}{550} = 70.4\,\text{ohm}$

Now, power factor cos $\phi = \frac{R}{Z}$ or R = Z cos ϕ $\therefore$ R = 70.4 × 0.8 = 56.32 ohm

Further, $Z^2 = R^2 + (\omega L)^2$ or $(\omega L) = \sqrt{(Z^2 - R^2)}$ Or $\omega L = \sqrt{(70.4)^2 - (56.32)^2} = 42.2\,\text{ohm}$

When the capacitor is connected in the circuit,

$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$ and cos $\phi = \frac{R}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}}$

when cos $\phi = 1, \omega L = \frac{1}{\omega C}$

$$\therefore \quad C = \frac{1}{\omega(\omega L)} = \frac{1}{2\pi f(\omega L)} = \frac{1}{(2 \times 3.14 \times 50) \times (42.2)} = 75 \times 10^{-6}\,F = 75\mu F.$$

17. $$\text{Irms} = \sqrt{\frac{1}{T/2}\int_0^{T/2} I^2 dt} \Rightarrow \text{Irms}^2 = \frac{2}{T}\int_0^{T/2} I^2 dt = \frac{2}{T}\int_0^{T/2}\left(\frac{2I_0 t}{T}\right)^2 .dt$$

$$\text{Irms} = \frac{8I_0^2}{T^3} \times \frac{T^3}{8 \times 3} = \frac{I_0^2}{3} \Rightarrow \text{Irms} = \frac{I_0}{\sqrt{3}}$$

PREVIOUS IIT JEE QUESTIONS

1. This is a problem of *L – C* oscillations

Charge stored in the capacitor oscillates simple harmonically as $Q = Q_0 \sin(\omega t \pm \phi)$

Here, Q_0 = maximum value of Q = 200 $\mu C = 2 \times 10^{-4}$ C

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2 \times 10^{-3} H)(5.0 \times 10^{-6} F)}} = 10^4\ s^{-1}$$

Let at $t = 0$, $Q = Q_0$ then ... (1)

$Q(t) = Q_{-0} \cos \omega t$

$I = \frac{dQ}{dt} = -Q_0\, \omega \sin \omega t$ and ...(2)

$\frac{dI}{dt} = -Q_0 \omega^2 \cos(\omega t)$ ...(3)

(a) *When* Q = 100 μC or $\frac{Q_0}{2}$, $\cos\omega t = \frac{1}{2}$

From Eq. (3) :

$$\left|\frac{dI}{dt}\right| = (2.0 \times 10^{-4}\ C)(10^{4}\ s^{-1})^{2}\left(\frac{1}{2}\right)$$

$$\left|\frac{dI}{dt}\right| = 10^{4}\ A/s$$

(b) *When* $Q = 200\ \mu C$ or Q_0 then $\cos(\omega t) = 1$ *i.e.,* $\omega t = 0, 2\pi$ ....

or $I = 0$

(c) $I_{max} = Q_0\omega = (2.0 \times 10^{-4}\ C)(10^{4}\ s^{-1})$

$I_{max} = 2.0$ A

(b) From energy conservation.

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}$$

or $Q = \sqrt{LC\,(I_{max}^2 - I^2)}$

$$I = \frac{I_{max}}{2} = 1.0\ A$$

$\therefore Q = \sqrt{(20\times10^{-3})(5.0\times10^{-6})(2^2 - I^2)}$

$Q = \sqrt{3} \times 10^{-4}\ C$

or $Q = 1.732 \times 10^{-4}\ C$

2. Current leads voltage in RC cobination.

$$\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR} = 1 \quad \Rightarrow \phi = \frac{\pi}{4}$$

3. Inductive reactance

$X_L = \omega L = (50)(2\pi)(35 \times 10^{-3})$ » $11\ \Omega$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(11)^2 + (11)^2} = 11\sqrt{2}\ \Omega$

Given $v_{rms} = 220$ volt

Hence, amplitude of voltage $v_0 = \sqrt{2}\ v_{rms} = 220\sqrt{2}$ volt

$\therefore$ Amplitude of current $i_0 = \frac{v_0}{Z} = \frac{220\sqrt{2}}{11\sqrt{2}} = 20$ A

Phase difference f $= \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{11}{11}\right) = \frac{\pi}{4}$

In $L - R$ circuit voltage leads the current. Hence, instantaneous current in the circuit is,

$i = (20A)\sin(\omega t - \pi/4)$

Corresponding $i - t$ graph is shown in figure.